Set D

1). Solution:-

i) let α and β be the two root of equation
$ax^2+bx+c=0$

$\alpha+\beta = -\frac{b}{a}$ and $\alpha.\beta = \frac{c}{a}$

$\alpha-\beta = (\alpha+\beta)^2 - 4\alpha\beta = \left(\frac{-b}{a}\right)^2 - \frac{4c}{a}$

$= \frac{b^2}{a^2} - \frac{4c}{a}$

Now, The root of the new equation are $(\alpha-\beta)^2$ and $(\alpha+\beta)^2$

$(\alpha-\beta)^2 \times (\alpha+\beta)^2 = \left(\frac{b^2}{a^2} - \frac{4c}{a}\right) \times \left(\frac{-b}{a}\right)^2$

$= \frac{(b^2-4ac)}{a^2} \times \frac{b^2}{a^2}$

$(\alpha-\beta)^2 + (\alpha+\beta)^2 = \frac{b^2}{a^2} - \frac{4a}{a} + \frac{b^2}{a^2}$

$= \frac{b^2-4a^2+b^2}{a^2}$

Equation: x^2 - (Sum of root)x + product of root

$x^2 - \left(\frac{b^2-4ac+b^2}{a^2}\right)x + \left(\frac{(b^2-4a^2)+b^2}{a^4}\right)$

$a^4x^2 - xa^2(2b^2-4ac) + b^2(b^2-4ac) = 0$

$a^4x^2 - 2xa^2(b^2-2ac) + b^2(b^2-4ac) = 0$

is the required quadratic equation

ii) According to question

$\alpha+\beta = \alpha^2+\beta^2$

$\alpha+\beta = (\alpha+\beta)^2 - 2\alpha\beta$

$\frac{-b}{a} = \left(\frac{-b}{a}\right)^2 - \frac{2c}{a}$

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$$\frac{-b}{a} = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$\frac{-b}{a} = \cancel{\frac{b^2}{b}} = \frac{b^2 - 2ac}{a^2}$$

$$-ba = b^2 - 2ac$$

$$2ac = ab + b^2$$ proved.

Set D

17. If α and β are two roots of equation $ax^2+bx+c=0$ find the equation whose roots are $(\alpha-\beta)^2$ and $(\alpha+\beta)^2$. Also, If sum of roots of the equation be equal to the sum of their squares, show $2ac = ab+b^2$.

Given equation is $ax^2+bx+c=0$ Then,

$$\alpha+\beta = \frac{-b}{a}, \quad \alpha\cdot\beta = \frac{c}{a}$$

Now, Since the root of required equation are $(\alpha-\beta)^2$ and $(\alpha-\beta)^2$

Sum of roots $= (\alpha-\beta)^2 + (\alpha+\beta)^2$

$$\alpha^2 - 2\alpha\beta + \beta^2 + \alpha^2 + 2\alpha\beta + \beta^2$$

$$= 2\alpha^2 + 2\beta^2$$

$$= 2(\alpha^2+\beta^2)$$

$$= 2\left(\frac{-b}{\alpha}\right)^2 - 2\cdot\frac{c}{\alpha}$$

$$= \frac{2b^2 - 2bc}{\alpha^2}$$

Product of root $= (\alpha-\beta)^2\cdot(\alpha+\beta)^2$

$$= \frac{b^2}{b^4}(b^2-4\alpha c)$$

The equation is $x^2 - 2\left\{\left(\frac{-b}{\alpha}\right)^2 - \frac{2c}{\alpha}\right\}x$

$$+ \frac{b^2}{a^2}(b^2-4ac) = 0$$

$$\alpha^4x^2 - 2\alpha^2(b^2-2\alpha c)x + b^2(b^2-4\alpha c) = 0$$

Now,
let α and β be the roots of given quadratic equation
Sum of the roots ie. $\alpha+\beta = -\frac{b}{a}$

Product of roots ie. $\alpha\beta = \frac{c}{a}$

It is given that
Sum of the roots = Sum of squares of the roots

ie. $\frac{-b}{a} = \alpha^2+\beta^2$

$$\frac{-b}{a} = (\alpha+\beta)^2 - 2\alpha\beta$$

$$\frac{-b}{a} = \left(\frac{-b}{a}\right)^2 - \frac{2c}{a}$$

$$-ab = b^2 - 2ac$$
$$ab + b^2 = 2ac$$

13a. Using matrix method solve the system of equations
$x+2y+z = -5$, $2x-y+z = 6$, $x-y+3z = -3$

Given equation are
$x+2y-z = -5$ —— ①
$2x-y+z = 6$ —— ②
$x-y-3z = -3$ —— ③
Writing in matrix form
$AX = C$
$X = A^{-1}C$ —— ④

13 a.

Solution:

By using matrix method

coeff. x	y	z	constant
1	2	-1	-5
2	-1	1	6
1	-1	-3	-3

$$D = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 1 & -1 & -3 \end{vmatrix} = 1\begin{vmatrix} -1 & 1 \\ -1 & -3 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}$$

$= 4 - 2\times(-7) - 1\times(-1)$

$= 4 + 14 + 1$

$= 19$

$$D_x = \begin{vmatrix} -5 & 2 & -1 \\ 6 & -1 & 1 \\ -3 & -1 & -3 \end{vmatrix} = -5\begin{vmatrix} -1 & 1 \\ -1 & -3 \end{vmatrix} - 2\begin{vmatrix} 6 & 1 \\ -3 & -3 \end{vmatrix} - 1\begin{vmatrix} 6 & -1 \\ -3 & -1 \end{vmatrix}$$

$= -5\times4 - 2\times(-15) - 1\times(-9)$

$= -20 + 30 + 9$

$= 19$

$$D_y = \begin{vmatrix} 1 & -5 & -1 \\ 2 & 6 & 1 \\ 1 & -3 & -3 \end{vmatrix} = 1\begin{vmatrix} 6 & 1 \\ -3 & -3 \end{vmatrix} + 5\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & 6 \\ 1 & -3 \end{vmatrix}$$

$= -15 + 5\times(-7) - 1(-12)$

$= -15 - 35 + 12$

$= -38$

$$D_z = \begin{vmatrix} 1 & 2 & -5 \\ 2 & -1 & 6 \\ 1 & -1 & -3 \end{vmatrix} = 1\begin{vmatrix} -1 & 6 \\ -1 & -3 \end{vmatrix} - 2\begin{vmatrix} 2 & 6 \\ 1 & -3 \end{vmatrix} - 5\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}$$

$= 9 - 2(-12) - 5\times(-1)$

$= 9 + 24 + 5$

$= 78$

$x = \frac{D_1}{D} = \frac{19}{19} = 1$

$y = \frac{D_2}{D} = \frac{-38}{19} = -2$

$z = \frac{D_3}{D} = \frac{38}{19} = 2$

Hence the required value of x, y, z are 1, -2, 2

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Where, $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 1 & -1 & -3 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$C = \begin{pmatrix} -5 \\ 6 \\ -3 \end{pmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 1 & -1 & -3 \end{vmatrix}$

$1\begin{vmatrix} -1 & 1 \\ -1 & -3 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} + (-1)\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}$

$1(3+1) - 2(-7) - 1(-1)$

$= 2+14+1$

$= 17 \neq 0$ So, A^{-1} exists

cofactor of $1 = \begin{vmatrix} -1 & 1 \\ -1 & -3 \end{vmatrix} = 4$

cofactor of $2 = -\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} = 7$

cofactor of $-1 = \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -1$

cofactor of $2 = -\begin{vmatrix} 2 & -1 \\ -1 & -3 \end{vmatrix} = 7$

cofactor of $-1 = \begin{vmatrix} 1 & -1 \\ 1 & -3 \end{vmatrix} = -2$

cofactor of $1 = -\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = 3$

Cofactor of $1 = \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 1$

Cofactor of $-1 = -\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = -2$

Cofactor of $-1 = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -5$

Cofactor (matrix) $= \begin{pmatrix} 4 & 7 & -1 \\ 7 & -2 & 3 \\ 1 & -2 & -5 \end{pmatrix}$

So,

Ad. of $A = \begin{pmatrix} 4 & 7 & 1 \\ 7 & -2 & -2 \\ -1 & 3 & -5 \end{pmatrix}$

$A^{-1} = \frac{1}{17} \begin{pmatrix} 4 & 7 & 1 \\ 7 & -2 & -2 \\ -1 & 3 & -5 \end{pmatrix}$

$X = A^{-1} C$

$X = \frac{1}{17} \begin{pmatrix} 4 & 7 & 1 \\ 7 & -2 & -2 \\ -1 & 3 & -5 \end{pmatrix} \begin{pmatrix} -5 \\ 1 \\ -3 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{17} \begin{pmatrix} -5\times 4 + 7\times 6 + 1\times -3 \\ 7\times -5 + 6\times -2 + (-3)\times(-2) \\ -5\times -1 + 6\times 3 + (-3)\times(-5) \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{19}{17} \\ \frac{-41}{17} \\ \frac{38}{17} \end{pmatrix}$

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b. Define group. If (G, o) is a group then show $(aob)^{-1} = b^{-1} o a^{-1}$

Let G be a non-empty set and $*$ is a binary operation on G. Then the algebraic structure $(G, *)$ is said to be group if the operation $*$ satisfies the following axioms

(G_1) Closure Axiom. G is closed under the operation $*$.
ie. $a*b \in G$ for all $a, b \in G$

(G_2) Associative Axiom. The binary operation $*$ is associative
ie. $(a*b)*c = a*(b*c)$ for all $a, b, c \in G$

(G_3) Identity Axiom. There exist an element $e \in G$ such that
$a*e = a = e*a$ for all $a \in G$
The element e is called the identity of 'a' with respect to '$*$' in G.

(G_4) Inverse Axiom Each element of G posseses inverse, ie. for each element $a \in G$, there exist an element $b \in G$, such that $a*b = e = b*a$.

We have

$(a o b) o (b^{-1} o a^{-1})$

$((a o b) o b^{-1}) o a^{-1}$ [$\because$ associative law]

$a o (b o b^{-1}) o a^{-1}$ [by associative law]

$(a o e) o a^{-1}$ [By inverse law]

$a o a^{-1}$ [identity law]

e [inverse law]

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$(a \circ b)^{-1} = b^{-1} \circ a^{-1}$ proved.

Q4. Prove that $4(\cot^{-1} 3 + \operatorname{cosec}^{-1} \sqrt{5}) = \pi$

L.H.S

$$4(\cot^{-1} 3 + \operatorname{cosec}^{-1} \sqrt{5})$$

$$4\left(\tan^{-1}\frac{1}{3} + \operatorname{cosec}^{-1}\sqrt{5}\right) \quad -(1)$$

let $\operatorname{cosec}^{-1}\sqrt{5} = A$

$$\operatorname{cosec} A = \frac{\sqrt{5}}{1} = \frac{h}{p}$$

$$b = \sqrt{(-\sqrt{5})^2 - (1)^2} = 2$$

$$\tan A = \frac{p}{b} = \frac{1}{2}$$

$$A = \tan^{-1}\frac{1}{2}$$

$$\operatorname{cosec}^{-1}\sqrt{5} = \tan^{-1}\frac{1}{2}$$

from (1)

$$4\left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right)$$

$$4\tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3}\cdot\frac{1}{2}}\right)$$

$$4\tan^{-1}(1)$$

$$4 \times \frac{\pi}{4} = \pi \text{ proved.}$$

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b. Find the equation of the ellipse whose latus rectum is half the major axis and focus is at (3,0)

For ellipse

Focus $=(3,0) = (\pm a, 0)$ $\therefore a = \pm 3$.

$\therefore a^2 = 9$

latus rectum: $\frac{2b^2}{a} = \frac{2a}{2}$ [half of major axis]

$$2b^2 = a^2$$

$$2b^2 = (\pm 3)^2$$

$$b^2 = \frac{9}{2}$$

Now, equation of ellipse is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{x^2}{9} - \frac{y^2}{9/2} = 1$$

$x^2 - 2y^2 = 9$ is required equation

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15a. Calculate the correlation coefficient from the following data

x	10	12	14	20	22
y	8	9	7	14	13

	x	y	x^2	y^2	$x \cdot y$
	10	8	100	64	80
	12	9	144	81	108
	14	7	196	49	98
	20	14	400	196	280
	22	13	484	169	286
Total	78	51	1324	559	852

$n = 5$, $\Sigma x = 78$, $\Sigma y = 51$, $\Sigma x^2 = 1324$,
$\Sigma y^2 = 559$, $\Sigma x \cdot y = 852$

$$\bar{x} = \frac{78}{5} = 15.6$$

$$\bar{y} = \frac{51}{5} = 10.2$$

$$r = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{5 \times 852 - 78 \times 51}{\sqrt{5 \times 1324 - (78)^2}\sqrt{5 \times 559 - (51)^2}}$$

$$\frac{4260 - 3978}{2\sqrt{134}\sqrt{194}}$$

$$= 0.87$$

#

6. 20% of the bulbs produced by a machine are non-defective. In the sample of 4 bulbs determine the probability of getting at least one bulb are defective.

Solution:

n = no of bulb = 4

p = prob. of a defective bulb = 20% = 0.2

q = prob. of a non-defective bulb = 1-0.2 = 0.8

$$p(r) = C(n,r)p^r q^{n-r}$$

p(0) = 0.8×4 = 3.2

p(1) = 4×0.8×0.2 = 0.64

1 - P(0) - P(1)

1 - 3.2 - 0.64

= -2.84

16. Use Simplex method to solve the following LPP maximize (w) = 5x + 3y

Subject to the constraints: 2x + y ≤ 40, x + 2y ≤ 50; x, y ≥ 0

Introduce r and s be the non-negative slack variable.

$2x + y + r = 40$

$x + 2y + s = 50$

Now, The standard form of given LP is

$2x + y + r + 0.s + 0.w = 40$

$x + 2y + 0r + s + 0.w = 50$

$-5x - 3y + 0.r + 0.s + w = 0$

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Initial simplex tableau.

B.V	x	y	r	s	w	R.H.S
r	2	1	1	0	0	40
s	1	2	0	1	0	50
	-5	-3	0	0	1	0

Since -5 is the most negative entry so C_1 is the pivot column and $\frac{40}{2} < \frac{50}{1}$ so, 2 is pivot element.

Apply $R_1 \to \frac{R_1}{2}$

	x	y	r	s	w	R.H.S
x	1	1/2	1/2	0	0	20
s	1	2	0	1	0	50
	-5	-3	0	0	1	0

Apply $R_2 : R_2 - R_1$ and $R_3 = 5R_1 + R_3$

	x	y	r	s	w	R.H.S
x	1	1/2	1/2	0	0	20
s	0	3/2	-1/2	1	0	30
	0	-1/2	1/2	0	1	100

↑

Again C_2 is the pivot column and since $\frac{30}{3/2} < \frac{20}{1/2}$ So, $\frac{3}{2}$ is the pivot element

Apply: $R_2 : \frac{2}{3} R_2$

	x	y	r	s	w	R.H.S
x	1	1/2	1/2	0	0	20
y	0	1	-1/3	2/3	0	20
	0	-1/2	1/2	0	1	100

Since all the value are positive so, it has optimal solution $w = 100$ at $x = 10$ and $y = 20$

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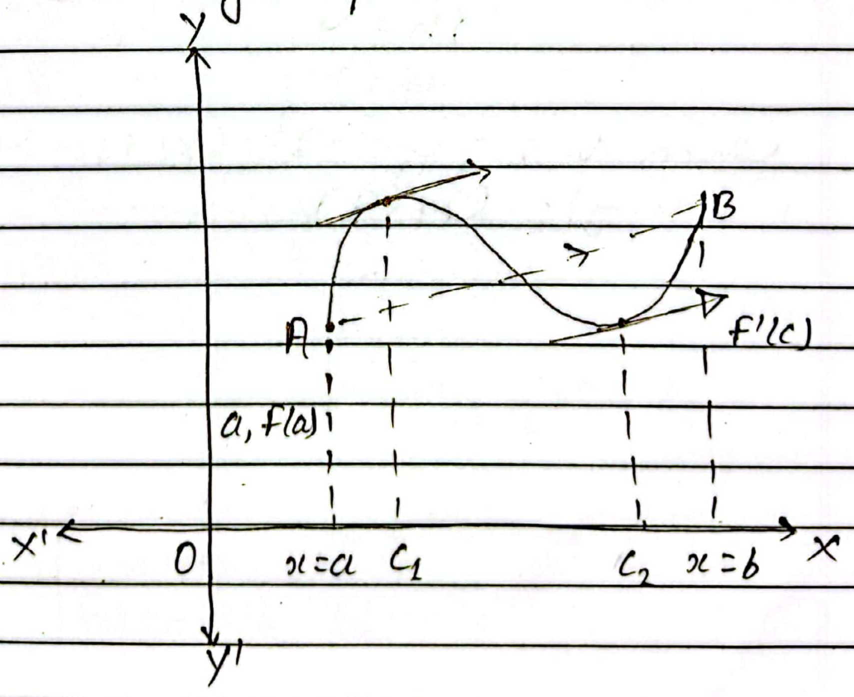
17. State first mean value theorem. Interpret the statement geometrically. Using the theorem, find the point on the curve $f(x) = x^2 - 6x + 1$ at which tangent is parallel to the chord joining the points $(1,-4)$ and $(3,-8)$.

First mean value theorem state that
If the function $f(x)$ is
(i) continuous in closed interval $[a,b]$
(ii) differentiable in open interval (a,b)
then there exists at least a point $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

Geometrically intepretation:



Mean value theorem say that in the continuous function, it has tangent at energy point then there exist at least a point on the curve which tangent in parallel to the second joining the end points of the curve.

Solution:

let (α, β) be the point on the given curve

$F(x): y = x^2 - 6x + 1$ —— (i)

$\beta = \alpha^2 - 6\alpha + 1$ —— (ii)

from (i) $\frac{dy}{dx} = 2x - 6$

$\frac{dy}{dx}$ at $(\alpha, \beta) = 2\alpha - 6$

Slope of twopoint Joining Chord is

$= \frac{-8+4}{3-1} = \frac{-4}{2} = -2$

Since They are parallel so,

$2x - 6 = -2$

$2x = 6 - 2$

$2x = 4$

$x = 2$

When $x = 2$, $y = 2^2 - 6 \times 2 + 1 = 4 - 12 + 1 = -7$

$(x, y) = (2, -7)$

18. Prove $\int \operatorname{cosec} x\,dx$

Solution:

$$\int \operatorname{cosec} x\,dx = \log\left(\tan\frac{x}{2}\right)+c$$

$$\int \operatorname{cosec} x\,dx = \int \frac{1}{\sin x}\,dx = \int \frac{dx}{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}$$

$$\frac{1}{2}\int \frac{1}{\sin\frac{x}{2}\cdot\cos\frac{x}{2}} \times \frac{\sec^2\frac{x}{2}}{\sec^2\frac{x}{2}}\,dx$$

$$=\frac{1}{2}\int \frac{\sec^2 x/2}{\tan x/2}$$

put $\tan\frac{x}{2} = y$

$$2\,dy = \sec^2\frac{x}{2}\,dx$$

$$I = \frac{1}{2}\int \frac{2\,dy}{y}$$

$$= \log y + c$$

$$= \log\left(\tan\frac{1}{2}x\right)+c \text{ proved}$$

$$\int \frac{dx}{2\sin x+3\cos x}$$

put $r\cos\theta = 2$ and $r\sin\theta = 3$

$$r^2 = 2^2+3^2 = 4+9 = 13$$

$$r = \sqrt{13}$$

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$\tan\theta = \frac{y}{x} = \frac{3}{2}$ $\therefore \theta = \tan^{-1}\left(\frac{3}{2}\right)$

Now,

$I = \frac{1}{r}\int \frac{dx}{\cos\theta\cdot\sin x + \sin\theta\cdot\cos x}$

$= \frac{1}{\sqrt{13}}\cdot\int \frac{dx}{\sin(x+\theta)}$

$\frac{1}{\sqrt{13}}\int \operatorname{cosec}(x+\theta)$

$\frac{1}{\sqrt{13}}\log\left(\tan\frac{(x+\theta)}{2}\right)$

$\frac{1}{\sqrt{13}}\log\left[\tan\frac{1}{2}\left(x+\tan^{-1}\left(\frac{3}{2}\right)\right)\right]$

19.

Solution:-

Demand function $Q = F(P) = 22500 - 75P$

i) $R = F(P) = Q\cdot P = (22500-75P)P = 22500P - 75P^2$

ii) $F(P) = -75P^2 + 22500P$

$F'(P) = -150P + 22500$

$F''(P) = -150$

Since $F''(P) < 0$, so it is concave downwards.

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iii) $TR = -75p^2 + 22500p$

At $p = 40$

$R = -75(40)^2 + 22500 \times 40$

$R = 780000$

iv) $Q = 22500 - 75p$

At $p = 40$

$Q = 22500 - 40 \times 75$

$Q = 19500$ unit

v) $\left(\frac{-b}{2a}, \frac{4cb^2}{4a}\right) = \left[\frac{-22500}{2 \times (-75)}, \frac{0-(22500)^2}{4 \times (-75)}\right]$

$(150, +1687500)$

Group 'C'

Solution

20b. let t_{r+1} be the general term of

$$\left(\frac{3x^2}{2} - \frac{1}{3x^2}\right)^{12}$$

where $n = 12$

$$t_{r+1} = (-1)^r\, C(n,r)\, p^{n-r} q^r$$

$$t_{r+1} = (-1)^r\, C(12,r) \left(\frac{3x^2}{2}\right)^{12-r} \left(\frac{1}{3x^2}\right)^r$$

$$(-1)^r\, C(12,r)\, \frac{(3x^2)^{12-r-r}}{2^{12-r}}$$

$$= (-1)^2\, C(12,r)\, \frac{3^{12-2r}}{2^{12-r}} \cdot x^{24-12r}$$

To get the term Independent of x

put $24-12r = 0$

$r = 2$

$\therefore\ t_{2+1} = t_3$ i.e. 3rd term is Independent

ii) Since, $12+1 = 13$ is the odd number then the middle term is given as $t_{\frac{n}{2}+1}$

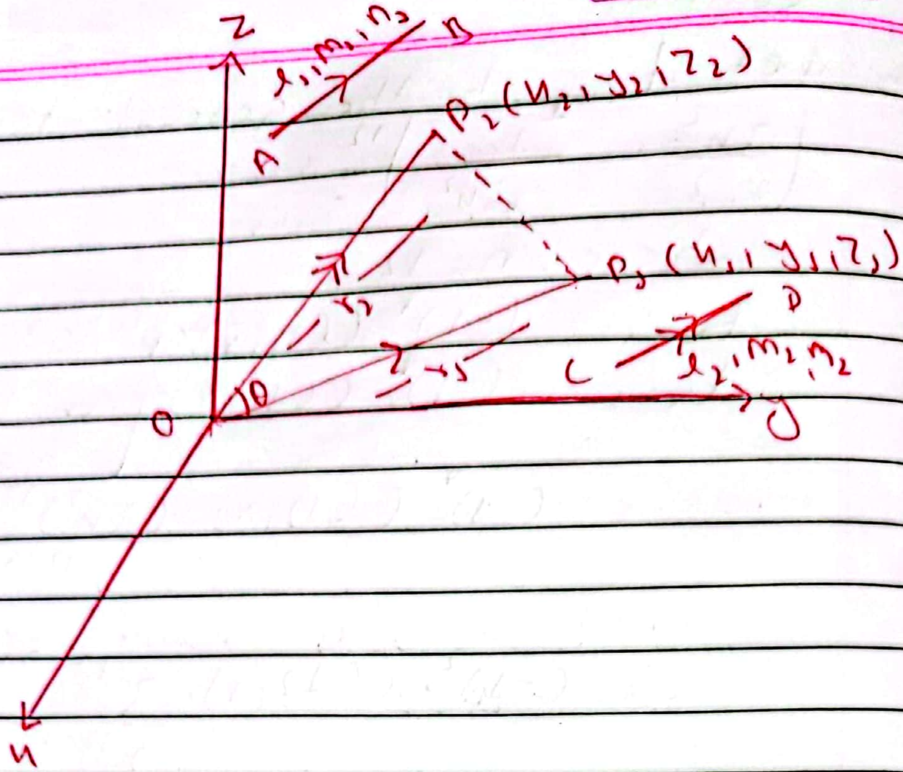
$$t_{\frac{12}{2}+1} = t_{6+1} = (-1)^6\, C(12,6) \left(\frac{3x^2}{2}\right)^{12-6} \left(\frac{1}{3x^2}\right)^6$$

$$= 924 \times \left(\frac{1}{2}\right)^6$$

$$= \frac{924}{64}$$

middle term $= \frac{231}{16}$

21.a.



let (l_1, m_1, n_1) and (l_2, m_2, n_2) be the direction cosines of two given lines AB and CD respectively

let OP_1 and OP_2 be the lines through origin O parallel to the line AB and CD respectively so that the angle between them is the same as the angle between the line AB and CD. Where θ be the angle

Also, the dcs of OP_1 and OP_2 are (l_1, m_1, n_1) and (l_2, m_2, n_2) respectively. let the coordinate of P_2 and P_1 is equal to (x_2, y_2, z_2) and (x_1, y_1, z_1) respectively

If $OP_2 = r_2$ the projection of OP_2 on OP_1 is $r_2 \cos\theta$. Also the

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projection of OP_2 on OP_1 is equal to $l_1 x_2 + m_1 y_2 + n_1 z_2$

Thus we have,

$$r_2 \cos\theta = l_1 x_2 + m_1 y_2 + n_1 z_2$$

$$\cos\theta = l_1 \frac{x_2}{r_2} + m_1 \frac{y_2}{r_2} + n_1 \frac{z_2}{r_2}$$

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

Which gives angle θ.

i) If two lines are parallel to each other then the angle between them is $\theta = 0$

$$\cos 0 = l_1 l_2 + m_1 \cdot m_2 + n_1 n_2$$

$$1 = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$1 + 1 = 2(l_1 l_2 + m_1 m_2 + n_1 \cdot n_2)$$

$$(l_1^2 + m_1^2 + n_1^2) + (l_2^2 + m_2^2 + n_2^2) - 2(l_1 l_2 + m_1 m_2 + n_1 n_2) = 0$$

$$(l_1 - l_2)^2 + (m_1 - m_2)^2 + (n_1 - n_2)^2 = 0$$

This is only possible if

$l_1 - l_2 = 0 \quad \therefore l_1 = l_2$

$m_1 - m_2 = 0 \quad \therefore m_1 = m_2$

$n_1 - n_2 = 0 \quad \therefore n_1 = n_2$

Hence the lines will be parallel to each other if only $l_1 = l_2$, $m_1 = m_2$ and $n_1 = n_2$

ii) perpendicular

→ If two lines are perpendicular than angle between them $\theta = 90$

$$\cos 90 = l_1 l_2 + m_1 m_2 + n_1 \cdot n_2$$

$$l_1 l_2 + m_1 \cdot m_2 + n_1 \cdot n_2 = 0$$

b. Solution:

let O be the position vector

$\vec{OA} = (1,4,6)$ and $\vec{OB} = (-2,5,1)$

we have,

$\vec{a} = (1,4,6)$ and $\vec{b} = (-2,5,-7)$

$\vec{a} \times \vec{b} = \begin{matrix} 1 & 4 & 6 & 1 & 4 \\ -2 & 5 & 1 & -2 & 5 \end{matrix}$

$= (4-30, -12-1, 5+8)$

$= (26, -13, 13)$

$|\vec{a} \times \vec{b}| = \sqrt{(26)^2 + (-13)^2 + (13)^2}$

$= 13\sqrt{6}$

Unit vector containing to the $\vec{OA}$ and $\vec{OB}$ which is perpendicular $= \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$= \dfrac{(26, -13, 13)}{13\sqrt{6}}$

$= \dfrac{26\vec{i} - 13\vec{j} + 13\vec{k}}{13\sqrt{6}}$

$= \dfrac{2\vec{i} - \vec{j} + \vec{k}}{\sqrt{6}}$

22a.

L-Hospital's Rule states that if f(x) and g(x) and also their derivatives f'(x) and g'(x) are continuous at x=a and if f(a) = g(a) = 0 then

$\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f'(x)}{\lim_{x \to a} g'(x)}$

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$$= \frac{f'(a)}{g'(a)}$$

provided that $g'(a) \neq 0$

Solution:

$$\lim_{x \to 0} \frac{x - \sin x \cdot \cos x}{x^3} \quad \left[\text{from } \frac{0}{0}\right]$$

$$\lim_{x \to 0} \frac{2x - 2\sin x \cos x}{2x^3}$$

$$\lim_{x \to 0} \frac{2 - 2\cos 2x}{6x^2}$$

$$\lim_{x \to 0} \frac{0 + 2 \times 2 \sin 2x}{12x}$$

$$\lim_{x \to 0} \frac{4 \times 2 \cos x}{12}$$

$$\cancel{\lim_{x \to 0}} \frac{8 \cos 0}{12}$$

$$= \frac{2}{3}$$

b. Solution

let $y = x^{\sin^2 \frac{x}{4}}$

Taking log on both side

$\log y = \sin^2 \frac{x}{4} \log x$

$$\frac{d \log y}{dy} = \frac{d(\sin^2 \frac{x}{4} \cdot \log x)}{dx}$$

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$$\frac{1}{y} \times \frac{dy}{dx} = \sinh^2 \frac{x}{a} \cdot \frac{d \log x}{dx} + \log x \, \frac{d \sinh^2 \frac{x}{a}}{d \sinh \frac{x}{a}} \times \frac{d \sinh \frac{x}{a}}{d \frac{x}{a}} \times \frac{d \frac{x}{a}}{dx}$$

$$\frac{1}{y} \times \frac{dy}{dx} = \sinh^2 \frac{x}{a} \times \frac{1}{x} + \log x \cdot 2 \sinh \frac{x}{a} \cosh \frac{x}{a} \times \frac{x}{a}$$

$$\frac{dy}{dx} = y \left[\sinh^2 \frac{x}{a} \times \frac{1}{x} + \log x \cdot \sin \frac{2hx}{a} \times \frac{1}{a} \right]$$

$$\frac{dy}{dx} = x^{\sinh^2 \frac{x}{a}} \left[\frac{1}{x} \sinh^2 \frac{x}{a} + \frac{\log x}{a} \sin \frac{2hx}{a} \right]$$

c. Solution:

$$(x^2+1) \frac{dy}{dx} + 2xy = 3x^2$$

$$\frac{dy}{dx} + \frac{2xy}{(1+x^2)} = \frac{3x^2}{(1+x^2)} \quad \text{——— (1)}$$

Comparing eqn (1) with $\frac{dy}{dx} + Py = Q$

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$\therefore P = \frac{2x}{(1+x^2)}$ and $Q = \frac{3x^2}{(1+x^2)}$

$\int P\,dx = \int \frac{2x}{1+x^2}\,dx = \log(1+x^2)$

$IF = e^{\int P\,dx} = e^{\log(1+x^2)} = (1+x^2)$

Now,

$$y \cdot (IF) = \int Q\,(IF)\,dx$$

$$y\,(1+x^2) = \int \frac{3x^2}{(1+x^2)} \times (1+x^2)\,dx$$

$$y\,(1+x^2) = \int 3x^2\,dx$$

$$= \frac{3x^3}{3} + C$$

$$y(1+x^2) = x^3 + C$$